



دفتر :

بحوث عمليات (1)

Operations Research 1

سالي عياش

للطالبة

اللجنة الأكاديمية لقسم الهندسة الصناعية

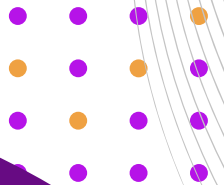
2025



Turbo IEG.Com



Turbo Team Youtube



Model Development

- A mathematical expression that describes the problem's objective is referred to as the **objective function**. For example, the profit equation $P=10x$ would be an objective function for a firm attempting to maximize profit. A production capacity constraint would be necessary if, for instant, 5 hours are required to produce each unit and only 40 hours are available per week. Let x indicate the number of units produced each week. The production time constraint is given by $5x \leq 40$
- The value of $5x$ is the total time required to produce x units; the symbol $\leq$ indicates that the production time required must be less than or equal to the 40 hours available.
- The decision problem or question is the following; How many units of the product should be scheduled each week to maximize profit? A complete mathematical model for this simple production problem is

Maximize $P=10x$	objective function
Subject to (s.t.)	
$5x \leq 40$	first constraint
$x \geq 0$	second constraint
- The $x \geq 0$ constraint requires the production quantity x to be greater than or equal to zero, which simply recognizes the fact that it is not possible to manufacture a negative number of units. The optimal solution to this model can be easily calculated and is given $x=8$, with an associated profit of \$80. **This model is an example of a linear programming model.**

No. CH1

Linear programming

Model → Constraints / objective Fun / variables
profit equation $P = 10x$ → For example
* profit Maximize / Minimize
objective Fun. → إذا في
non negativity constraint → ثابت لاي model

Question 1: A manufacturer Produces three products

Radio, Calculator, TV. Profit for each product:

15\$ For each unit of Radio

5\$ " " of Calculator

20\$ " " of TV

اول خطوة

① تعريف Variables

② تحديد objective Fun

③ Constraints

production data are as follows:

Products	Per unit		
	Material (Kgs)	Labor (Hours)	Machining (Hours)
Radio	2	1	1
calculator	1	0.5	0.75
TV	10	3	3
Total available per week	500	700	350

The manufacturer wishes to establish the weekly production plan which maximizes the profit. Formulate as a Linear programming model.

Sol:

x_1 = number of Radios produced Weekly
 x_2 = number of calculators produced Weekly
 x_3 = number of TV's produced Weekly

$$\text{Max } z = 15x_1 + 5x_2 + 20x_3$$

s.t

$$2x_1 + 1x_2 + 10x_3 \leq 500$$

$$x_1 + 0.5x_2 + 3x_3 \leq 700$$

$$x_1 + 0.75x_2 + 3x_3 \leq 350$$

$$x_1, x_2, x_3 \geq 0$$

ex 8 profit per unit

$$\text{Max } z = 7x_1 + 6x_2$$

s.t.

$$x_1 + x_2 \leq 4$$

$$x_1 + 2x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

نستعملها ما يكون

Solve :

منقربين فف

Use Graphical Method to solve LPM : Graphical

خطوات ① رسم Constraints

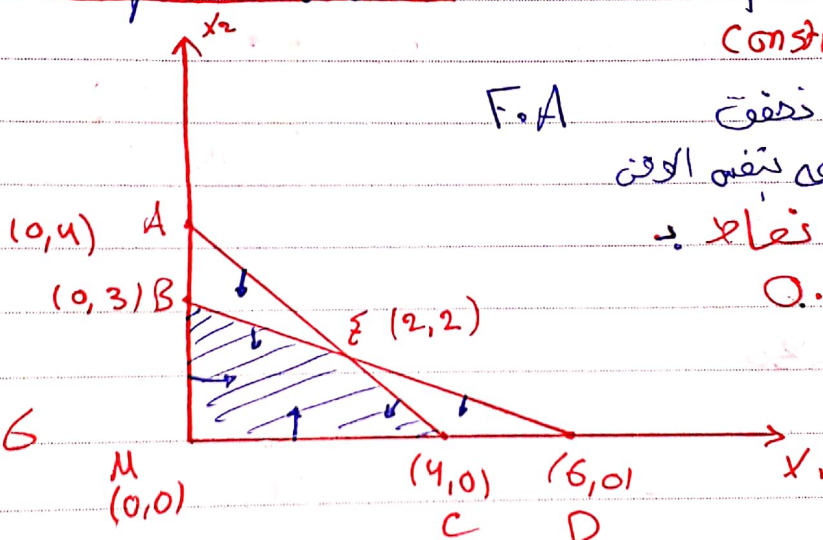
② تحديد area ذفق

جميع الشروط المحتملة بنفس الوقت

③ جعل وتكونه نفاذ

O. F

F.A



$$x_1 + x_2 = 4$$

$$x_1 + 2x_2 = 6$$

F.A : BECM

optimal ~ Max النقاط التي رخص (corners)

optimal sol

$$x_1 = 4 \text{ units}$$

$$x_2 = 0 \text{ units}$$

$$z = \$28 \text{ max profit}$$

Corners	$z = 7x_1 + 6x_2$
(0,0)	\$0
(0,3)	\$18
(2,2)	\$26
(4,0)	\$28

$$\begin{array}{r} 36 \\ 24 \\ \hline 120 \end{array}$$

ex

$$\text{Max } z = 10x_1 + 2x_2$$

s.t

$$12x_1 + 4x_2 \leq 24$$

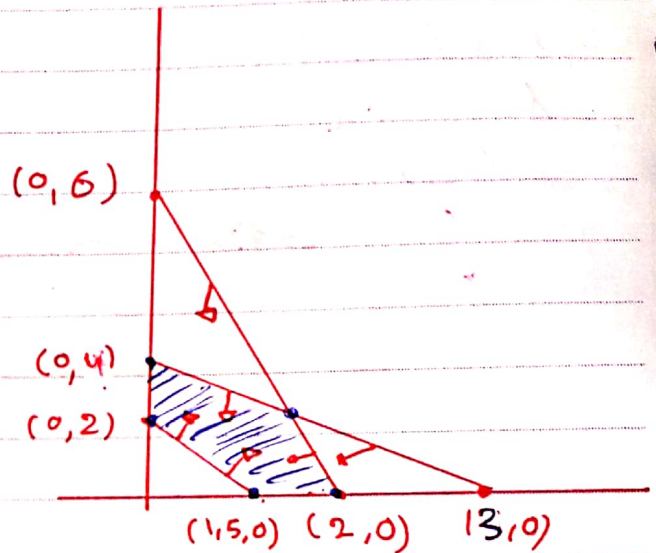
$$4x_1 + 3x_2 \leq 12$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1, x_2 \geq 0$$

$$-5x_2 = -12$$

$$12$$



Corners

$$z = 10x_1 + 2x_2$$

$$(0, 2)$$

$$\$4$$

$$(0, 4)$$

$$\$8$$

$$(1, 5)$$

$$\$150$$

$$(2, 0)$$

$$\$20$$

$$(1.5, 1.5)$$

$$\$22.8$$

$$12x_1 + 4x_2 \leq 24$$

$$4x_1 + 3x_2 \leq 12$$

Optimal sol

$$x_1 = 2 \text{ units}$$

$$x_2 = 0 \text{ units}$$

$$z = 20\$ \text{ max profit}$$

ex

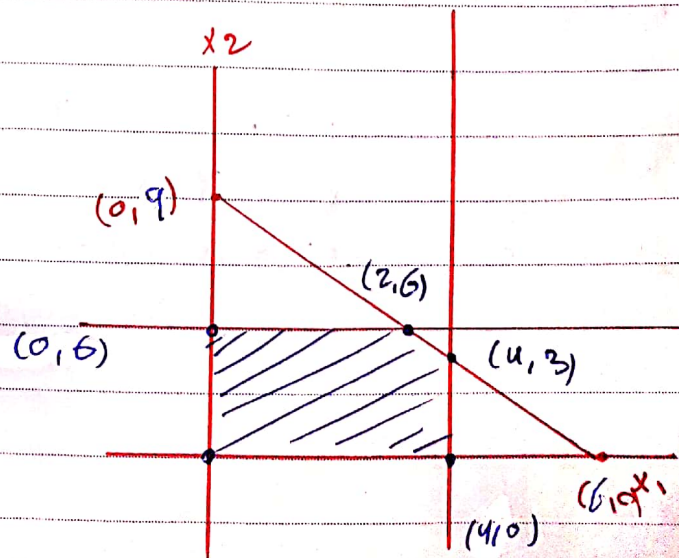
$$\text{Max } z = 3x_1 + 5x_2$$

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$



Corners	$Z = 3x_1 + 5x_2$
(0,0)	\$0
(0,6)	\$30
(2,6)	\$36
(4,3)	\$27
(4,0)	\$12

Optimal sol

 $x_1 = 2 \text{ units}$ $x_2 = 6 "$ $Z = \$36$

ex:

$$\text{Min } Z = 24x_1 + 28x_2$$

s.t

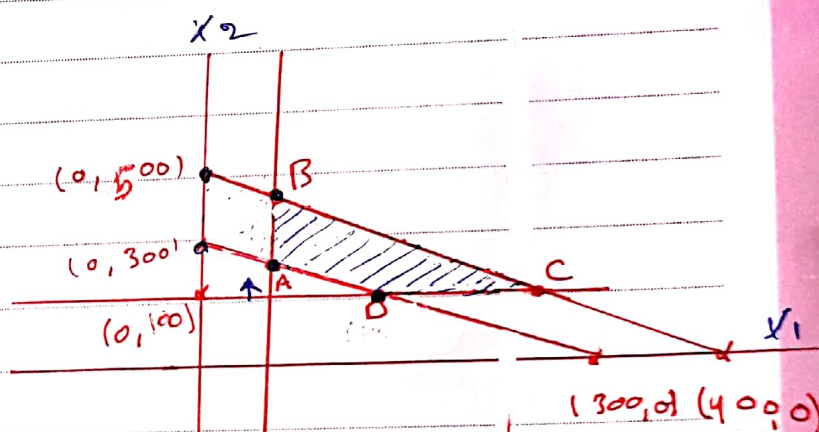
$$5x_1 + 4x_2 \leq 2000$$

$$x_1 \geq 80$$

$$x_1 + x_2 \geq 300$$

$$x_2 \geq 100$$

$$x_1, x_2 \geq 0$$



Corners	$Z = 24x_1 + 28x_2$
D (200, 100)	\$7600
C (320, 100)	\$10480
A (80, 220)	\$8080
B (80, 400)	\$13120

$$x_1 + x_2 = 300$$

$$-x_2 = 100$$

Optimal sol

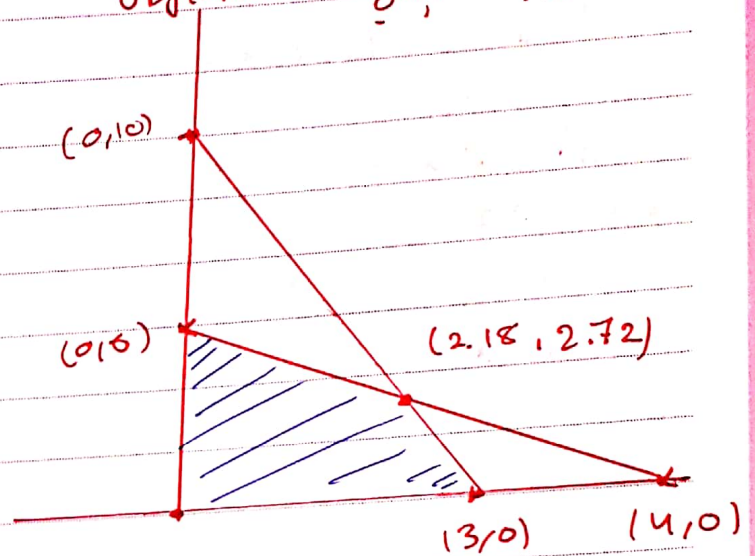
 $x_1 = 200 \text{ units}$ $x_2 = 100 "$ $Z = \$7600$

No. Special cases

1. Alternative optimal
 $\text{Max } Z = 3x_1 + 2x_2$
 s.t

$6x_1 + 4x_2 \leq 24$
 $10x_1 + 3x_2 \leq 30$
 $x_1, x_2 \geq 0$

corners	$Z = 3x_1 + 2x_2$
(0,6)	\$12
(3,0)	\$9
(0,0)	\$0
(2.18, 2.72)	\$12



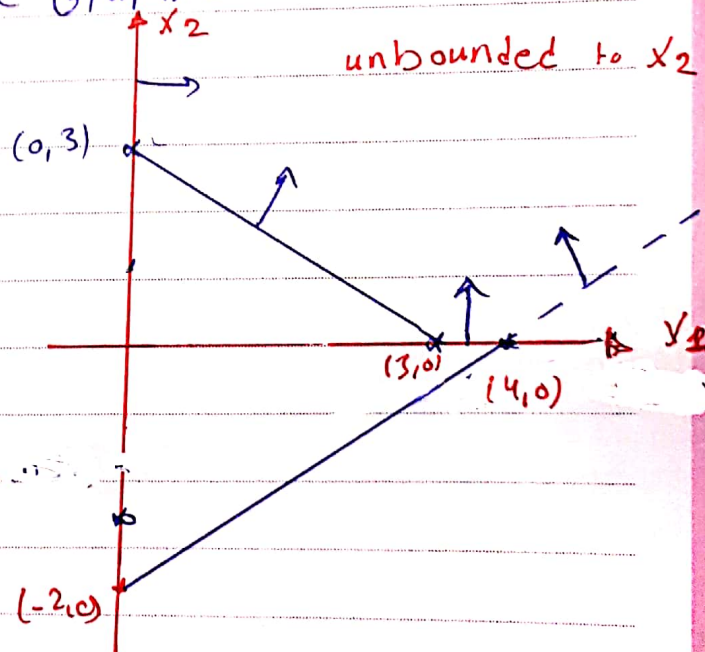
Optimal Sol
 1. $x_1 = 0$ unit
 $x_2 = 6$ units
 $Z = \$12$

2. $x_1 = 2.18$ units
 $x_2 = 2.72$ units
 $Z = \$12$

$\text{Max } Z = 2x_1 + 3x_2$
 s.t

$x_1 + x_2 \geq 3$
 $x_1 - 2x_2 \leq 4$
 $x_1, x_2 \geq 0$

Use Graphical



2. ↑ unbounded solution

Area open along x_1 and x_2

3. InFeasible problems

لا يعمل فيما زعقن
السرو

$$\text{Max } Z = 4x_1 + 3x_2$$

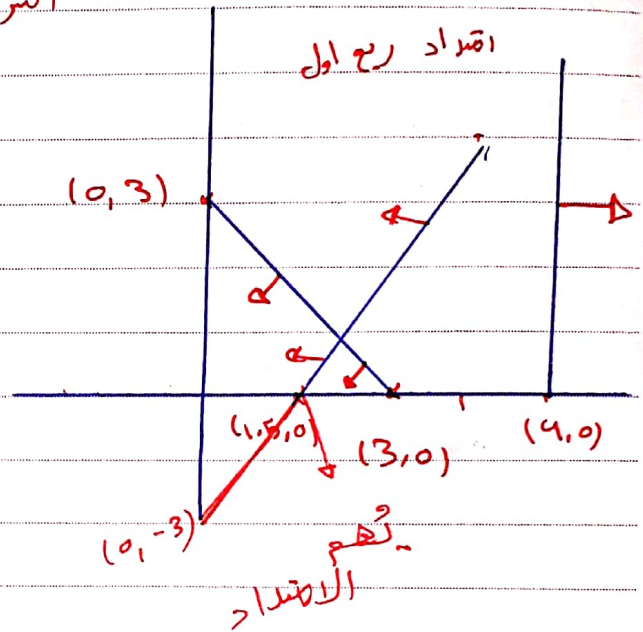
s.t

$$x_1 + x_2 \leq 3$$

$$2x_1 - x_2 \leq 3$$

$$x_1 \geq 4$$

$$x_1, x_2 \geq 0$$



4. Degeneracy - Redundant constraint.

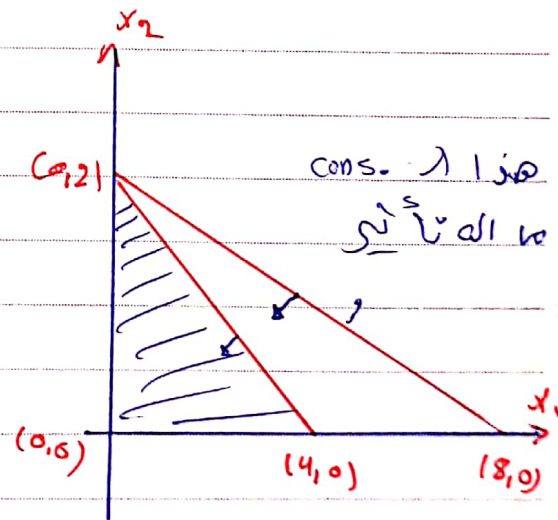
$$\text{Max } Z = 3x_1 + 9x_2$$

s.t

$$x_1 + 4x_2 \leq 8$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$



No. Sensitivity Analysis (Post-optimality)

Profit per unit
 $\text{Max } Z = 5x_1 + 7x_2$

s.t

$x_1 \leq 6$

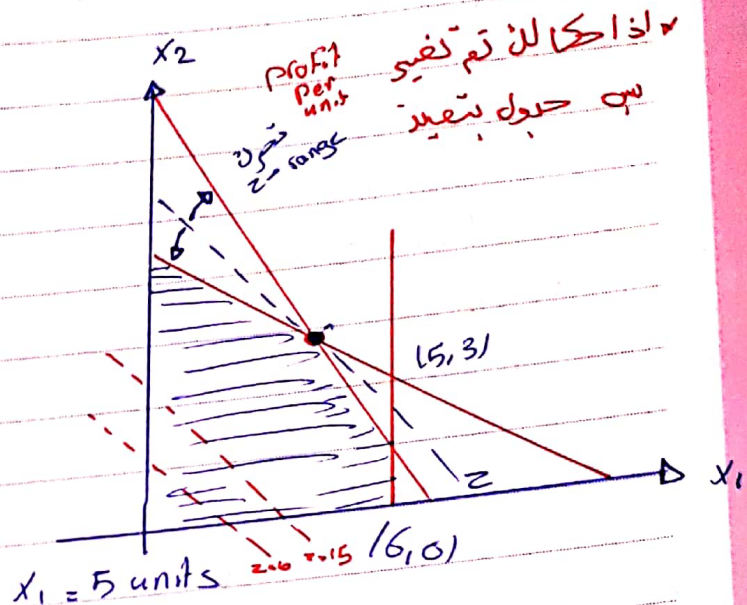
$2x_1 + 3x_2 \leq 19$

Binding Constraints $x_1 + x_2 \leq 8$

$x_1, x_2 \geq 0$

فرق

$10 = 5x_1 + 7x_2$
 $15 = 5x_1 + 7x_2$
حتى اعرف
الاجابة
زائدة



$x_1 = 5 \text{ units}$

$x_2 = 3$

$Z = \$ 46$

اذا كان تم تغيير
المعادلات
المحددة
المعادلات
المحددة

طريقة اخرى لإيجاد optimal
حتى يغيرها عند نقطة optimal

ex: solve

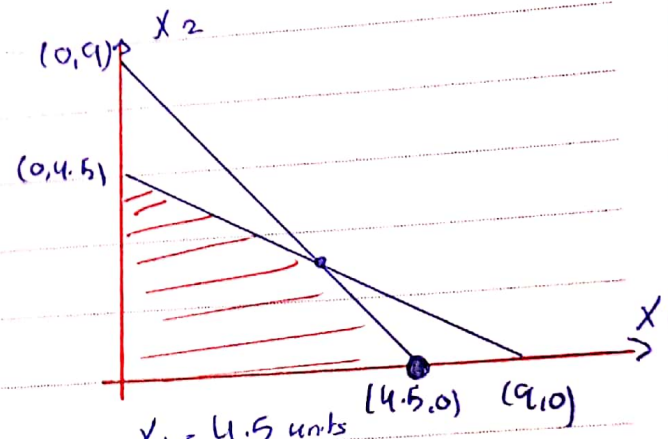
$\text{Max } Z = 3x_1 + x_2$

s.t

$2x_1 + x_2 \leq 9$

$x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$



$x_1 = 4.5 \text{ units}$

$x_2 = 0$

$Z = \$$

IF the optimal sol is on
the line then its Binding

$(4.5, 0)$

$2x_1 + x_2 = 9$

$2(4.5) + 0 = 9$

$9 = 9 \checkmark$

$x_1 + 2x_2 = 9$

$4.5 + 2(0) < 9$ not Binding

حتى نعرفها
طرف اليمنى = اليسرى
L.H.S = R.H.S

example

$$\text{Max } z = 3x_1 + 5x_2$$

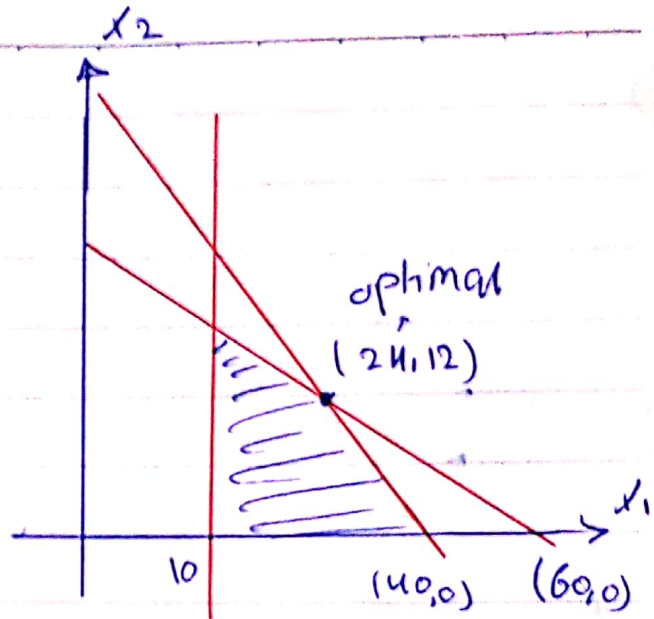
s.t

$$x_1 + 3x_2 \leq 60$$

$$3x_1 + 4x_2 \leq 120$$

$$x_1 \geq 10$$

$$x_1, x_2 \geq 0$$



$$\textcircled{1} x_1 + 3x_2 = 60$$

$$124 + 3(12) = 60 \quad \checkmark \quad \text{Binding}$$

$$\textcircled{2} 3(24) + 4(12) = 120$$

$$3x_1 + 4x_2 = 120$$

Range of optimality الإيجاد من القيمة الجاه

نفس اول سؤال تحت عنوان
sensitivity

$$\text{Max } z = 5x_1 + 7x_2$$

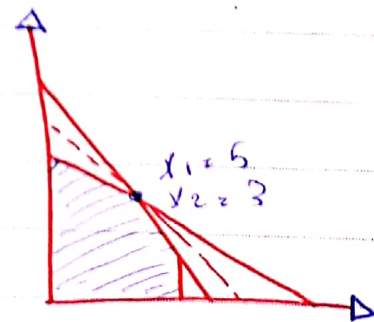
s.t

$$x_1 \leq 6$$

$$2x_1 + 3x_2 \geq 19$$

$$x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0$$



The limits of a range of optimality Found By changing
The slope of the objective function line within the limits
of the steps of the Binding constraint lines

= slope of an objective function line.

$$\text{Max } C_1 x_1 + C_2 x_2 \text{ is } -\frac{C_1}{C_2}$$

= slope of a constraint

$$a_1 x_1 + a_2 x_2 = b$$

$$-\frac{a_1}{a_2}$$

Range of optimality for C_1 : Coefficient 1 (5)

باختيار C_2 ثابت

Binding $\begin{cases} x_1 + x_2 = 8 & \text{slope } -1 \\ 2x_1 + 3x_2 = 19 & \text{slope } -\frac{2}{3} \end{cases}$

* Find Range of C_1 (with C_2 staying 7)

s.t the objective fun line slope constant lies between that of the two Binding

$$-1 \leq -\frac{C_1}{7} \leq -\frac{2}{3}$$

$$7 \geq C_1 \geq \frac{14}{3}$$

$$\frac{14}{3} \leq C_1 \leq 7$$

• Range of optimality for C_2 (C_1 constant) ₅

$$-1 \leq -\frac{5}{C_2} \leq -\frac{2}{3}$$

$$5 \leq C_2 \leq \frac{15}{2}$$

$$\text{Max } z = 2x_1 + 3x_2$$

s.t

$$3x_1 + 2x_2 \leq 6$$

$$-x_1 + x_2 \geq 0$$

$$x_1, x_2 \geq 0$$

$$3x_1 + 2x_2 = 6$$

$$\text{slope } -\frac{3}{2}$$

$$-x_1 + x_2 = 0$$

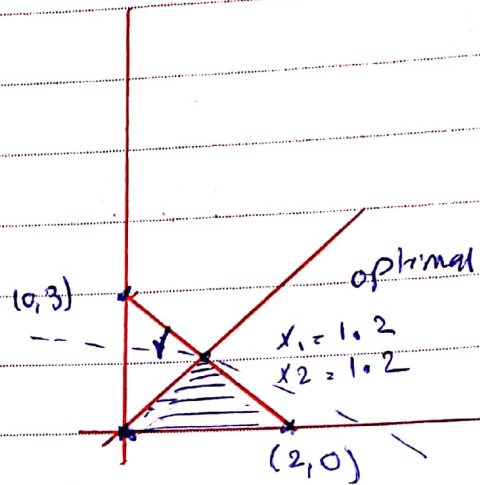
$$\text{slope } 1$$

$$-\frac{3}{2} \leq -\frac{C_1}{3} \leq 1$$

$$-\frac{3}{2} \leq -\frac{2}{C_2} \leq 1$$

$$-3 \leq C_1 \leq \frac{9}{2}$$

$$-2 \leq C_2 \leq \frac{4}{3}$$



CONSTRUCTION OF THE LP MODEL

This section illustrates the basic elements of an LP model by using a simple **two – variable** example. The results provide concrete ideas for the solution and interpretation of the general LP problem.

Example

Reddy Mikks produces both interior and exterior paints from two raw materials, *M1* and *M2*. The following table provides the basic data of the problem:

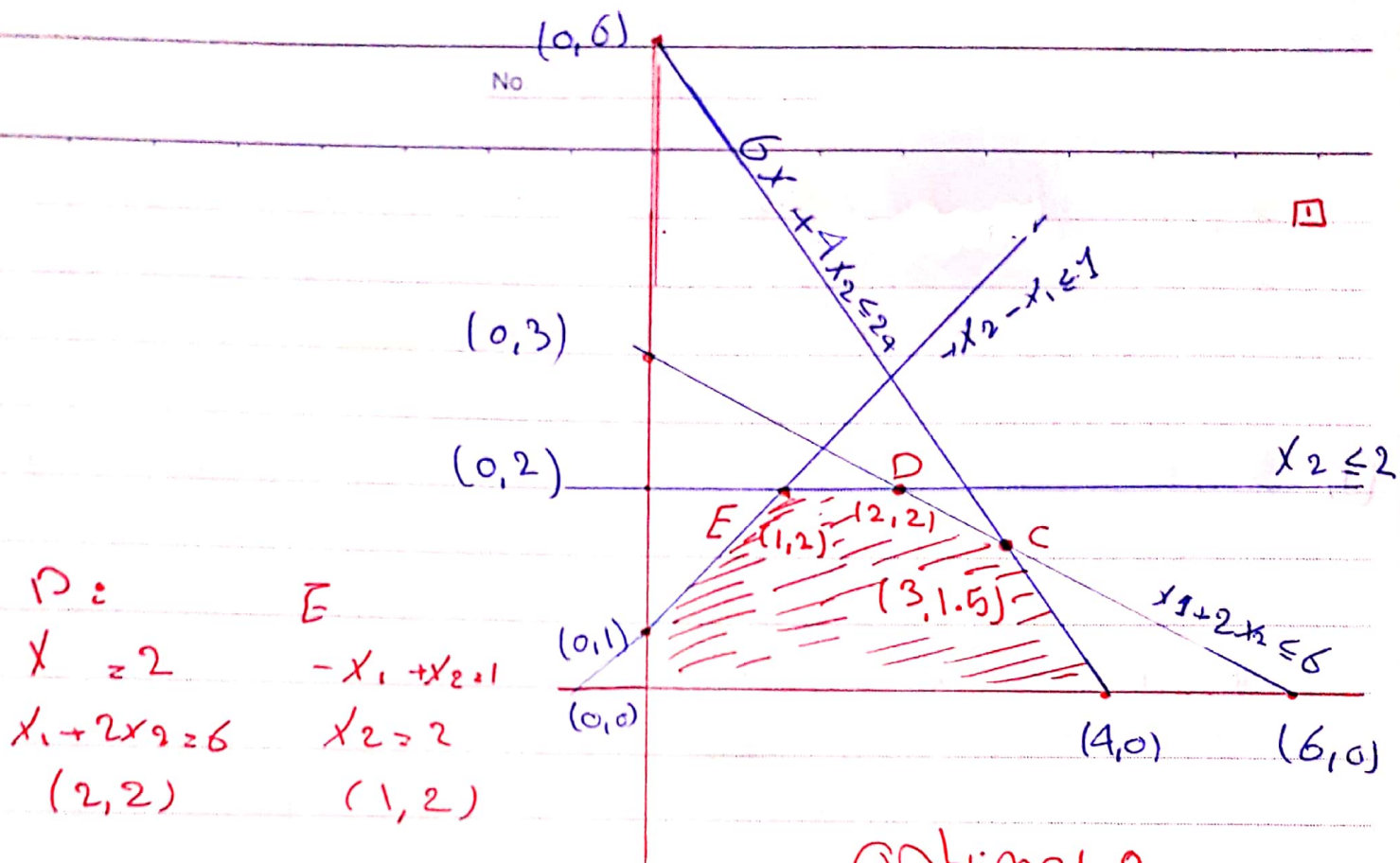
	<u>Tons of raw material per ton of</u>		<u>Maximum daily Availability (tons)</u>
	<u>Exterior paint</u>	<u>Interior paint</u>	
Raw material, <i>M1</i>	x_1 6	x_2 4	24
Raw material, <i>M2</i>	1	2	6
Profit per ton(\$1000)	5	4	

A market survey restricts the maximum daily demand of interior paint to 2 tons. Additionally, the daily demand of interior paint cannot exceed that of exterior paint by more than 1 ton.

Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit.

$$\begin{aligned}
 6x_1 + 4x_2 &\leq 24 \\
 x_1 + 2x_2 &\leq 6 \\
 x_2 &\leq 2 \\
 x_2 - x_1 &\leq 1 \\
 x_1, x_2 &\geq 0
 \end{aligned}
 \quad \left| \quad \begin{aligned} \max z &= 5x_1 + 4x_2 \end{aligned} \right.$$

- any LP model :
- ① Decision variables, we seek to determine
 - ② Objective (goal) we aim to optimize
 - ③ Constraints that we need to satisfy



Optimal :
 $x_1 = 2$ units
 $x_2 = 1.5$ units
 $z = \$21,000$

* ممکن ہے سوال دیکھ کر الحل دیکھ کر
 پرفٹ پر unit
 پھر جدول میں

Range of optimality

$x_1 + 2x_2 \leq 6$ slope $-\frac{1}{2}$
 $6x_1 + 4x_2 \leq 24$ slope $-\frac{6}{4}$

$$-\frac{1}{2} \leq -\frac{C_1}{4} \leq -\frac{3}{2}$$

x_1 : type 1
 x_2 : type 2

h_1 : labor type 1
 time

h_2 : labor type 2
 time

2-10 A company produces two types of cowboy hats. Each hat of the first type requires twice as much labor time as does each hat of the second type. If all hats are of the second type, only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second types to 150 and 200 hats. Assume that the profit per hat is \$8 for type 1 and \$5 for type 2. Determine the number of hats of each type to produce to maximize profit.

Max $z = 8x_1 + 5x_2$

$x_1, x_2 \geq 0$

as h_2 is

~~$x_1 \leq 150$~~ $x_1 \leq 150$

~~$x_2 \leq 200$~~ $x_2 \leq 200$

$h_1 x_1 + h_2 x_2 = 500 h_2$

$2x_1 + x_2 = 500$

$\frac{h_1}{h_2} = 2$

$h_1 = 2h_2$

$A = x_1$

$B = x_2$

Max $z = 20x_1 + 50x_2$

$x_2 \leq 100$

$2x_1 + 4x_2 \leq 240$

$x_1, x_2 \geq 0$

$\frac{x_1}{x_1 + x_2} \geq 0.8$

2 A company produces two products, A and B. The sales volume for A is at least 80% of the total sales of both A and B. However, the company cannot sell more than 100 unit of A per day. Both products use one raw material whose maximum daily availability is limited to 240 lb a day. The usage rates of the raw material are 2 lb per unit of A and 4 lb per unit of B. The unit prices for A and B are \$20 and \$50, respectively.

(a) Determine the optimal product mix for the company.

solve

تقسيم فكرة
 الجدول

8. Dean's Furniture Company assembles from precut lumber two types of kitchen cabinets: regular and deluxe. The regular cabinets are painted white, and the deluxe ones are varnished. Both the painting and the varnishing occur in one department. The daily capacity of the assembly department

can produce a maximum of 200 regular cabinets and 150 deluxe ones. Varnishing a deluxe unit takes twice as much time as painting a regular one. If the painting/varnishing department is dedicated to the deluxe units only, it can complete 180 units daily. The company estimates that the profits per unit for the regular and deluxe cabinets are \$100 and \$140, respectively.

- (a) Formulate the problem as a linear program and find the optimal production schedule per day.
- (b) Suppose that because of competition, the profits per unit of the regular and deluxe units must be reduced to \$80 and \$110, respectively. Use sensitivity analysis to determine whether or not the optimum solution in (a) remains unchanged.

ضروري حل سؤال 2-10 simplex على Min / Max

Simplex Method: 2 variables and more

① Standard Form → تحويل الاكبر الى اصغر او اشارة سيادي

ex

$$6x_1 + 3x_2 - 2x_3 \leq 50$$

* add a slack variable → لا يكون اصغر

$$6x_1 + 3x_2 - 2x_3 + x_4 = 50$$

objective fun لا يوجد له تأثير

actual amount

$$x_4 = 50 - (6x_1 + 3x_2 - 2x_3)$$

x_4 measures the unused amount 50 - actual amount

ex

$$4x_1 + 2x_2 + 3x_3 \geq 15 \rightarrow \text{لا يكون اكبر}$$

* subtract a surplus variable

$$4x_1 + 2x_2 + 3x_3 - x_4 = 15$$

$$x_4 = (4x_1 + 2x_2 + 3x_3) - 15$$

ex:

$$\text{Max } Z = 80x_1 + 120x_2$$

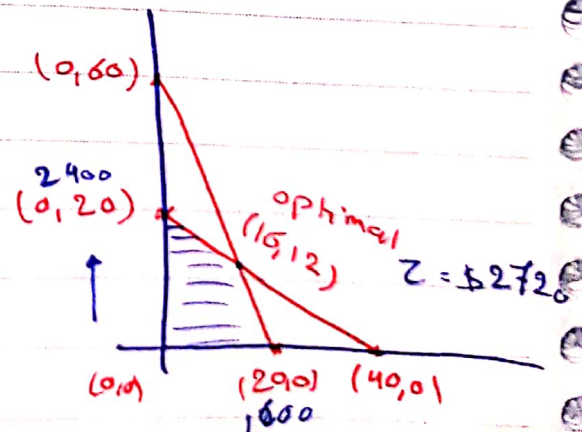
s.t

$$2x_1 + 4x_2 \leq 80$$

$$3x_1 + x_2 \leq 60$$

$$x_1, x_2 \geq 0$$

$$\begin{aligned} 2x_1 + 4x_2 &= 80 \\ -12x_1 - 4x_2 &= -240 \\ -10x_1 &= -160 \\ x_1 &= 16 \end{aligned}$$



طريقة جرافيك

Simplex Method

No.

عدد معادلات

$$\text{Max } Z = 80x_1 + 120x_2 + 0x_3 + 0x_4$$

s.t

$$2x_1 + 4x_2 + x_3 + 0x_4 = 80$$

$$3x_1 + x_2 + 0x_3 + x_4 = 60$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$Z - 80x_1 - 120x_2 = 0$$

	x_1	x_2 entering	x_3	x_4	R.H.s	Ratio
Z	-80	-120	0	0		
x_3	2	4	1	0	80	20
x_4	3	1	0	1	60	60
	-20	0	30	0	2400	
x_2	$\frac{1}{2}$	1	$\frac{1}{4}$	0	20	
	2.5	0	$-\frac{1}{4}$	1	40	
	0	0	28	8	2720	
	0	1	$\frac{3}{10}$	$-\frac{1}{5}$	12	
	1	0	$-\frac{1}{10}$	$\frac{2}{5}$	16	

ممكن ما تكون مرتبه / اكثر من 3x3

نقل معادلات الى الجدول

بكر معادله لازم Identity

عنه عدد متفرات

بكر معادله x_3, x_4 row

Zero = Coeff

Max ← entering Most negative

min ← entering Most positive

less ratio ← leaving

Optimal

$$x_1 = 16$$

$$x_2 = 12$$

$$x_3 = 0$$

$$x_4 = 0$$

$$Z = 2720$$

كل ايتريشن يجب التأكد اذا Optimal اول

بما انو المسألة Max وفي negative row

not optimal

No.

linear Algebra

طريقة

فرق بين

$$2x_1 + 4x_2 + x_3 = 80$$

$$3x_1 + x_2 + x_4 = 60$$

①

$$x_1 = x_2 = 0$$

$$x_3 = 60$$

$$x_4 = 60$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 60 \\ 60 \end{bmatrix}$$

②

$$x_1 = x_3 = 0$$

$$x_2 = 20$$

$$x_4 = 40$$

$$\begin{bmatrix} 0 \\ 20 \\ 0 \\ 40 \end{bmatrix}$$

③

$$x_1 = x_4 = 0$$

$$x_2 = 60$$

$$x_3 = -160$$

$$\begin{bmatrix} 0 \\ 60 \\ -160 \\ 0 \end{bmatrix}$$

④

$$x_3 = x_4 = 0$$

$$x_1 = 16$$

$$x_2 = 12$$

عدد جابج

$$\frac{n!}{m!(n-m)!} = \frac{4!}{2!(4-2)!} = 6$$

عدد مولات

⑤

$$x_2 = x_3 = 0$$

$$x_1 = 40$$

$$x_4 = -60$$

⑥

$$x_2 = x_4 = 0$$

$$x_1 = 20$$

$$x_3 = 40$$

No. Solve

$$\text{Max } Z = 12x_1 + 8x_2$$

s.t.

$$5x_1 + 2x_2 \leq 150$$

$$2x_1 + 3x_2 \leq 100$$

$$4x_1 + 2x_2 \leq 80$$

$$x_1, x_2 \geq 0$$

Simplex

$$\text{Max } Z = 12x_1 + 8x_2$$

$$5x_1 + 2x_2 + x_3 = 150$$

$$2x_1 + 3x_2 + x_4 = 100$$

$$4x_1 + 2x_2 + x_5 = 80$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

$$Z - 12x_1 - 8x_2 = 0$$

No.

	x_1	x_2	x_3	x_4	x_5	R.H.S	ratio
Z	-12	-8	0	0	0	0	
x_3	5	2	1	0	0	150	30
x_4	2	3	0	1	0	100	50
$\rightarrow x_5$	4	2	0	0	1	80	20
	0	-2	0	0	3	240	min ratio
x_3	0	$-\frac{1}{2}$	1	0	$-\frac{5}{4}$	50	- لا يجوز
x_4	0	2	0	1	$\frac{1}{2}$	60	30 - فتوجه بالعدد الأصغر
Pivot - x_2	1	$\frac{1}{2}$	0	0	$\frac{1}{4}$	20	40
	0	0	0	1	$\frac{5}{2}$	300	
x_3	0	0	1	$\frac{1}{4}$	$-\frac{11}{8}$	65	
Pivot x_2	0	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	30	
x_1	1	0	0	$-\frac{1}{4}$	$\frac{3}{8}$	5	

Optimal sol

$$x_1 = 5$$

$$x_2 = 30$$

$$x_3 = 65$$

$$x_4 = 0$$

$$x_5 = 0$$

$$Z = \$ 300$$

$$\begin{aligned}
 x_3 &= 150 - (5x_1 + 2x_2) \\
 &= 150 - (5(5) + 2(30)) \\
 &\quad \text{actual} \\
 &\quad 85 \\
 &= 65
 \end{aligned}$$

القيمة

تقدم عذرجہ اکی Big M (penalty technique) او یساری

$$\min Z = 4x_1 + x_2$$

s.t

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

ماذا کو میں خراب دھ

Max - Min

ماذا بدست یی

Solve simplex

$$\min Z = 4x_1 + x_2 + M P_1 + M P_2$$

s.t

Max / Min افافہ

$M(P_1 + P_2)$

حل مشکل عدم وجود Identity

$$3x_1 + x_2 + P_1 = 3$$

$$4x_1 + 3x_2 - x_3 + P_2 = 6$$

$$x_1 + 2x_2 + x_4 = 4$$

$$= 3$$

to create

Identity

R.H.S ratio

	x_1	x_2	x_3	x_4
Z	-4	-1	0	0
	3	1	0	0
	4	3	-1	0
	1	2	0	1

لا یوجد Identity

	x_1	x_2	x_3	x_4	R_1	R_2	RHS
					$-M$	$-M$	0
z	-4	-1	0	0			
	3	1	0	0	1	0	3
	4	3	-1	0	0	1	6
	1	2	0	1	0	0	4

امثالاً يجب سؤال Basic variables
 zero row $\rightarrow$ ملاحظة z row $\rightarrow$ zero

متكامل Identity أدخلت لكن

هل المشكلة

$$R_1 = 3 - 3x_1 - x_2$$

$$R_2 = 6 - 4x_1 - 3x_2 + x_3$$

$$R_1 + R_2 = 9 - 7x_1 - 4x_2 + x_3$$

$$z = 4x_1 + x_2 + M(9 - 7x_1 - 4x_2 + x_3)$$

$$z + (7M - 4)x_1 + (4M - 1)x_2 - Mx_3 = 9M$$

No

Most positive

	x_1	x_2	x_3	x_4	R_1	R_2	R.H.S	ratio
Z	$7M-4$	$M-1$	M	0	0	0	$9M$	
R_1	3	1	0	0	1	0	3	3
R_2	4	3	-1	0	0	1	6	1.5
x_4	1	2	0	1	0	0	4	4
	0	$\frac{5}{3}M + \frac{1}{3}$	$-M$	0	$-\frac{1}{3}M + \frac{1}{3}$	0	$2M + 4$	
x_1	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	1	3
R_2	0	$\frac{5}{3}$	-1	0	$-\frac{4}{3}$	1	2	$\frac{6}{5}$
x_4	0	$\frac{5}{3}$	0	1	$-\frac{1}{3}$	0	3	$\frac{9}{5}$
	0	0	$\frac{1}{5}$	0	$-\frac{1}{5}M + \frac{8}{5}$	$-\frac{1}{5}$	$\frac{18}{5}$	
x_1	1	0	$\frac{1}{5}$	0	$\frac{3}{5}$	$-\frac{1}{5}$	$\frac{3}{5}$	3
x_2	0	1	$-\frac{3}{5}$	0	$-\frac{4}{5}$	$\frac{3}{5}$	$\frac{6}{5}$	—
x_4	0	0	1	1	1	-1	1	1
	0	0	0	$-\frac{1}{5}$	$\frac{7}{5}M$	$-M$	$\frac{17}{5}$	
x_1	1	0	0	$-\frac{1}{5}$	$\frac{2}{5}$	0	$\frac{2}{5}$	
x_2	0	1	0	$\frac{3}{5}$	$-\frac{1}{5}$	0	$\frac{9}{5}$	
x_3	0	0	1	1	1	-1	1	

Min ratio

Z row most negative

Optimal sol

$x_1 = \frac{2}{5} \text{ units}$

$x_2 = \frac{9}{5} \text{ "}$

$Z = \frac{17}{5}$

Two phase Method

$$\min Z = 4x_1 + x_2$$

s.t

$$3x_1 + x_2 \geq 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

$$\min Z = 4x_1 + x_2$$

s.t

min problem is $\geq$ phase 1

$$3x_1 + x_2 + R_1 = 3$$

$$4x_1 + 3x_2 - x_3 + R_2 = 6$$

$$x_1 + 2x_2 + x_4 = 4$$

$$x_1, x_2, x_3, x_4, R_1, R_2 \geq 0$$

$$\min r_2 \in R$$

$$R_1 = 3 - 3x_1 - x_2$$

$$R_2 = 6 - 4x_1 - 3x_2 + x_3$$

$$r_2 = R_1 + R_2 = 9 - 7x_1 - 4x_2 + x_3$$

$$r - 9 + 7x_1 + 4x_2 - x_3 = 0$$

$$r + 7x_1 + 4x_2 - x_3 = 9$$

No.

	x_1	x_2	x_3	x_4	P_1	P_2	R.H.S	
r	7	4	-1	0	0	0	9	
P_1	3	1	0	0	1	0	3	1
P_2	4	3	-1	0	0	1	6	1.5
x_4	1	2	0	1	0	0	4	4
r	0	5/3	-1	0	-7/3	0	2	
x_1	1	1/3	0	0	1/3	0	1	3
P_2	0	5/3	-1	0	-4/3	1	2	6/5
x_4	0	5/3	0	1	-1/3	0	3	9/5
r	0	0	0	0	-1	-1	0	
x_1	1	0	1/5	0	3/5	-1/5	3/5	
x_2	0	1	-3/5	0	-4/5	3/5	6/5	
x_4	0	0	1	1	1	-1	1	
end of phase								row Basic row Positive zero row

phase 2

$$Z = 4x_1 + x_2$$

$$x_1 + \frac{1}{5}x_3 = \frac{3}{5}$$

$$x_2 - \frac{3}{5}x_3 = \frac{6}{5}$$

$$Z = 4\left(\frac{3}{5} - \frac{1}{5}x_3\right) + \frac{6}{5} + \frac{3}{5}x_2$$

$$Z + \frac{1}{5}x_3 = \frac{18}{5}$$

ارجاع الاعداد min

row operation

	x_1	x_2	x_3	x_4	R.H	
z	0	0	$1/5$	0	$18/5$	
x_1	1	0	$1/5$	0	$3/5$	3
x_2	0	1	$-3/5$	0	$6/5$	—
x_4	0	0	1	1	1	1
z	0	0	0	$-1/5$	$17/5$	
x_1	1	0	0	$-1/5$	$2/5$	
x_2	0	1	0	$3/5$	$9/5$	
x_3	0	0	1	1	1	

optimal
 $x_1 = 2/5$
 $x_2 = 9/5$
 $z = 17/5$

$$\text{Max } z = 2x_1 + 5x_2$$

s.t

$$3x_1 + 2x_2 \geq 6$$

$$2x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

$$3x_1 + 2x_2 - x_3 + R_1 = 6$$

$$2x_1 + x_2 + x_4 = 2$$

$$x_1, x_2, R_1, x_3, x_4 \geq 0$$

$$\max R = 6 + x_3 - 2x_2 - 3x_1$$

special
Cases

$$x - x_3 + 2x_2 + 3x_1 = 6$$

	x_1	x_2	x_3	x_4	R	R.H.s	
r	3	2	-1	0	0	6	
R	3	2	-1	0	1	6	2
x_4	2	1	0	1	0	2	1
r	0	1/2	-1	-3/2	0	3	
R	0	1/2	-1	-3/2	1	3	6
x_1	1	1/2	0	1/2	0	1	2
r	-1	0	-1	-2	0	2	
R	-1	0	-1	-2	1	2	
x_2	2	1	0	1	0	2	

end of phase
* special case

① Infeasible →

کل شروع نہ ہو سکتے ہیں
لان R.H.s کی زیادہ سے زیادہ zero

ex:

$$\text{Max } z = 3x_1 + 2x_2$$

$$6x_1 + 4x_2 \leq 24$$

$$10x_1 + 3x_2 \leq 30$$

$$x_1, x_2 \geq 0$$

Solve:

Don't use Graphical

standard form $\text{Max } z = 3x_1 + 2x_2$

$$\text{s.t. } 6x_1 + 4x_2 + x_3 = 24$$

$$10x_1 + 3x_2 + x_4 = 30$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$z - 3x_1 - 2x_2 = 0$$

	x_1	x_2	x_3	x_4	R.H.S	P
z	-3	-2	0	0	0	
x_3	6	4	1	0	24	4
x_4	10	3	0	1	30	3
z	0	$-11/10$	0	$3/10$	9	
x_3	0	$22/10$	1	$-6/10$	6	$30/11$
x_1	1	$3/10$	0	$1/10$	3	LG
z	0	0	$11/22$	0	12	وهو افضل جواب
x_2	0	1	$10/22$	$-6/10$	$30/11$	alternative
x_1	1	0	$-3/22$	$2/11$	$24/11$	
z	0	0	$11/22$	0	12	
x_2	$66/4$	1	$21/44$	0	6	
x_4	$11/2$	0	$17/20$	1	12	

Alternative case

optimal $z = 125$

$$\begin{array}{l} x_1 = 24/11 \quad x_2 = 6 \\ x_2 = 30/11 \quad x_4 = 12 \end{array}$$

Max $z = 3x_1 + 9x_2$

s.t

$$x_1 + 4x_2 \leq 8$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Standard Form

$$z = 3x_1 + 9x_2$$

$$x_1 + 4x_2 + x_3 = 8$$

$$x_1 + 2x_2 + x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$z - 3x_1 - 9x_2 = 0$$

	x_1	x_2	x_3	x_4	RHS	Ratio
z	-3	-9	0	0	0	
x_3	1	4	1	0	8	
x_4	1	2	0	1	4	2 $\rightarrow$ Tie
z	-3/4	0	9/4	0	18	
x_2	1/4	1	1/4	0	2	
x_4	1/2	0	-1/2	1	0	
z	0	0	3/2	3/2	18	
x_2	0	1	1/2	-1/2	2	
x_1	1	0	-1	2	0	

اي ايشين قضا Tie
نتيجه بعده optimal
next

Redundant Case

$$\text{Max } Z = 2x_1 + x_2$$

s.t

$$x_1 - x_2 \leq 10$$

$$2x_1 \leq 40$$

$$x_1, x_2 \geq 0$$

$$\text{Max } Z = 2x_1 + x_2$$

s.t

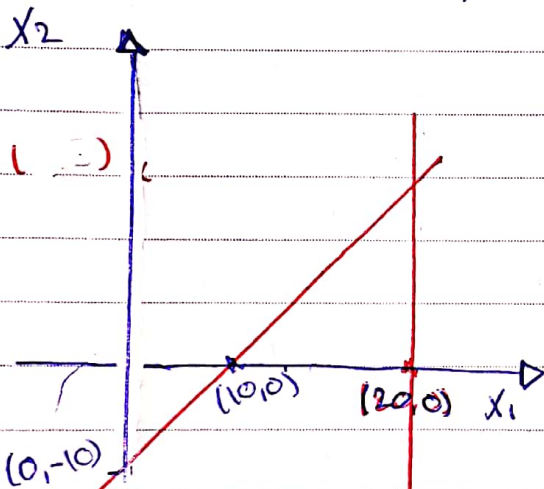
$$x_1 - x_2 + x_3 = 10$$

$$2x_1 + x_4 = 40$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$Z - 2x_1 - x_2 \geq 0$$

unbounded x_2
Graphical



Non positive
zero

non basic
variable

اذا كان x_2 غير
* unbounded
Case

ايش نكتب
نكون

اي

	x_1	x_2	x_3	x_4	R.H.S
Z	-2	-1	0	0	0
x_3	1	-1	1	0	10
x_4	2	0	0	1	40